

Family Dependence in $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models

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Abstract

Using experimental results at the Z -pole and atomic parity violation, we perform a χ^2 fit at 95% CL to obtain family-dependent bounds to Z_2 mass and $Z_\mu - Z'_\mu$ mixing angle θ in the framework of $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models. The allowed regions depend on the assignment of the quark families in mass eigenstates into the three different families in weak eigenstates that cancel anomalies.

1 Introduction

A very common alternative to solve some of the problems of the standard model (SM) consists on enlarging the gauge symmetry. For instance, the $SU(5)$ grand unification model of Georgi and Glashow [1] can unify the interactions and predicts the electric charge quantization; while the group E_6 can also unifies the interactions and might explain the masses of the neutrinos [2]. Nevertheless, such models cannot explain the origin of the fermion families. Some models with larger symmetries address this problem [3, 4]. A very interesting alternative to explain the origin of generations comes from the cancellation of chiral anomalies [5]. In particular, models with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 331 models, arise as a possible solution to this puzzle, where the three families are required in order to cancel chiral anomalies completely. An additional motivation to study these kind of models comes from the fact that they can also predict the charge quantization for a three family model even when neutrino masses are added [6].

Although cancellation of anomalies leads to some required conditions [7], such criterion alone still permits an infinite number of 331 models. In these models, the electric charge is defined in general as a linear combination of the diagonal generators of the group

$$Q = T_3 + \beta T_8 + XI. \quad (1)$$

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As it has been studied in the literature [7, 8], the value of the β parameter determines the fermion assignment, and more specifically, the electric charges of the exotic spectrum. Hence, it is customary to use this quantum number to classify the different 331 models. If we want to avoid exotic charges we are led to only two different models i.e. $\beta = \pm 1/\sqrt{3}$ [7, 9].

It has been recently obtained in ref [10] constraints to 331 models by examining the scalar sector. In summary, these constraints are obtained by requiring gauge invariance in the Yukawa sector and finding the possible vacuum alignment structures that respect the symmetry breaking pattern and provides the fermions and gauge bosons of the SM with the appropriate masses. By applying gauge invariance to the Yukawa lagrangian it is found that the Higgs bosons should lie in either a triplet, antitriplet, singlet or sextet representation of $SU(3)_L$. On the other hand, cancellation of chiral anomalies demands that the number of fermionic triplets and antitriplets must be equal [11]. Moreover, assuming the symmetry breaking pattern $SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$, we see that one triplet is necessary for the first symmetry breaking and two triplets for the second in order to give mass to the up and down quark sector. In some cases it is necessary to introduce a scalar sextet to give masses to the neutrinos [8].

The group structure of these models leads, along with the SM neutral boson Z , to the prediction of an additional current associated with a new neutral boson Z' . Unlike Z -boson whose couplings are family independent and the weak interactions at low energy are of universal character, the couplings of Z' are different for the three families due to the $U(1)_X$ values to each of them. Through the $Z - Z'$ mixing it is possible to study the low energy deviations of the Z couplings to the SM families [8, 12, 13]. In the quark sector each 331-family in the weak basis can be assigned in three different ways into mass eigenstates. In this way in a phenomenological analysis, the allowed region associated with the $Z - Z'$ mixing angle and the physical mass M_{Z_2} of the extra neutral boson will depend on the family assignment to the mass states.

In this work we report a phenomenological study through a χ^2 fit at the Z -pole to find the allowed region for the mixing angle between the neutral gauge bosons $Z - Z'$ and the mass of the Z_2 boson at 95% CL for three different assignments of the quark families [14]. We take into account the two main versions of the 331 models from the literature [8, 9], which correspond to $\beta = -\sqrt{3}$ and $-\frac{1}{\sqrt{3}}$ respectively.

This paper is organized as follows. Section 2 is devoted to summarize the Fermion, Scalar and Vector boson representations. In section 3 we describe the neutral currents and the vector and axial vector couplings of the model. In section 4 we perform the χ^2 analysis at the Z -pole including atomic parity violation at 95% CL. Finally, section 5 contains our conclusions.

2 The 331 spectrum

The fermionic spectrum under $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ is shown in table 1 for three families with $\beta = -\frac{1}{\sqrt{3}}$ and $-\sqrt{3}$, where the first case contains the Long model [9] and the second contains the bilepton model proposed by Pisano, Pleitez and Frampton [8]. We recognize three different possibilities to assign the physical quarks in each family representation as it

Representation
$q_{mL} = \begin{pmatrix} d_m \\ -u_m \\ J_m^{-1/3} (-4/3) \end{pmatrix}_L : (\mathbf{3}, \mathbf{3}^*, 0 \ (-1/3))_{-1/\sqrt{3}} (-\sqrt{3})$
$d_{mR}, u_{mR}, J_{mR}^{-1/3} (-4/3) : \mathbf{1}$
$q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ J_3^{2/3} (5/3) \end{pmatrix}_L : (\mathbf{3}, \mathbf{3}, 1/3 \ (2/3))_{-1/\sqrt{3}} (-\sqrt{3})$
$u_{3R}, d_{3R}, J_{3R}^{2/3} (5/3) : \mathbf{1}$
$\ell_{jL} = \begin{pmatrix} \nu_j \\ e_j \\ E_j^0 (1) \end{pmatrix}_L : (\mathbf{3}, \mathbf{3}, -1/3 \ (0))_{-1/\sqrt{3}} (-\sqrt{3})$
$e_{jR}, E_{jR}^0 (1) : \mathbf{1}$

Table 1: Fermionic content for three generations with $\beta = -\frac{1}{\sqrt{3}}$ ($-\sqrt{3}$). The third component is written with its electric charge. We take $m = 1, 2$.

is shown in table 2. At low energy, the three models from table 2 are equivalent and there are not any phenomenological feature that allow us to detect differences between them. In fact, they must reduce to the SM which is an universal family model in $SU(2)_L$. However, through the couplings of the three families to the additional neutral current (Z') and the introduction of a mixing angle between Z and Z' it is possible to recognize differences among the three models at the electroweak scale. It is noted that although we write the spectrum in the weak basis in table 1, we can consider three realizations in the mass basis in table 2.

For the scalar sector, we introduce the triplet field χ with Vacuum Expectation Value (VEV) $\langle \chi \rangle^T = (0, 0, \nu_\chi)$, which induces the masses to the third fermionic components. In the second transition it is necessary to introduce two triplets ρ and η with VEV $\langle \rho \rangle^T = (0, \nu_\rho, 0)$ and $\langle \eta \rangle^T = (\nu_\eta, 0, 0)$ in order to give masses to the quarks of type up and down respectively.

In the gauge boson spectrum associated with the group $SU(3)_L \otimes U(1)_X$, we are just interested in the physical neutral sector that corresponds to the photon, Z and Z' , which are written in terms of the electroweak basis for $\beta = -\frac{1}{\sqrt{3}}$ and $-\sqrt{3}$ as [15]

$$\begin{aligned}
 A_\mu &= S_W W_\mu^3 + C_W \left(\beta T_W W_\mu^8 + \sqrt{1 - \beta^2 T_W^2} B_\mu \right), \\
 Z_\mu &= C_W W_\mu^3 - S_W \left(\beta T_W W_\mu^8 + \sqrt{1 - \beta^2 T_W^2} B_\mu \right), \\
 Z'_\mu &= -\sqrt{1 - \beta^2 T_W^2} W_\mu^8 + \beta T_W B_\mu,
 \end{aligned} \tag{2}$$

Representation A	Representation B	Representation C
$q_{mL} = \begin{pmatrix} d, s \\ -u, -c \\ J_1, J_2 \end{pmatrix}_L : \mathbf{3}^*$ $q_{3L} = \begin{pmatrix} t \\ b \\ J_3 \end{pmatrix}_L : \mathbf{3}$	$q_{mL} = \begin{pmatrix} d, b \\ -u, -t \\ J_1, J_3 \end{pmatrix}_L : \mathbf{3}^*$ $q_{3L} = \begin{pmatrix} c \\ s \\ J_2 \end{pmatrix}_L : \mathbf{3}$	$q_{mL} = \begin{pmatrix} s, b \\ -c, -t \\ J_2, J_3 \end{pmatrix}_L : \mathbf{3}^*$ $q_{3L} = \begin{pmatrix} u \\ d \\ J_1 \end{pmatrix}_L : \mathbf{3}$

Table 2: Three different assignments for the $SU(3)_L$ family representation of quarks

and with eigenvalues

$$M_{A_\mu}^2 = 0; \quad M_{Z_\mu}^2 \simeq \frac{g^2}{2} \left[\frac{3g^2 + 4g'^2}{3g^2 + g'^2} \right] (\nu_\rho^2 + \nu_\eta^2); \quad M_{Z'_\mu}^2 \simeq \frac{2[3g^2 + g'^2]}{9} \nu_\chi^2, \quad (3)$$

where the Weinberg angle is defined as

$$S_W = \sin \theta_W = \frac{g'}{\sqrt{g^2 + (1 + \beta^2) g'^2}}, \quad T_W = \tan \theta_W = \frac{g'}{\sqrt{g^2 + \beta^2 g'^2}} \quad (4)$$

and g, g' correspond to the coupling constants of the groups $SU(3)_L$ and $U(1)_X$ respectively. Further, a small mixing angle between the two neutral currents Z_μ and Z'_μ appears with the following mass eigenstates

$$\begin{aligned} Z_{1\mu} &= Z_\mu C_\theta + Z'_\mu S_\theta; \quad Z_{2\mu} = -Z_\mu S_\theta + Z'_\mu C_\theta; \\ \tan \theta &= \frac{1}{\Lambda + \sqrt{\Lambda^2 + 1}}; \quad \Lambda = \frac{-2S_W C_W^2 g'^2 \nu_\chi^2 + \frac{3}{2} S_W T_W^2 g^2 (\nu_\eta^2 + \nu_\rho^2)}{g g' T_W^2 [3\beta S_W^2 (\nu_\eta^2 + \nu_\rho^2) + C_W^2 (\nu_\eta^2 - \nu_\rho^2)]}. \end{aligned} \quad (5)$$

3 Neutral currents

Using the fermionic content from table 1, we obtain the neutral coupling for the SM fermions

$$\begin{aligned} \mathcal{L}^{NC} &= \sum_{j=1}^3 \left\{ \frac{g}{2C_W} \overline{Q}_j \gamma_\mu \left[2T_3 P_L - 2Q_{Q_j} S_W^2 \right] Q_j Z^\mu \right. \\ &+ \frac{g}{2C_W} \overline{\ell}_j \gamma_\mu \left[2T_3 P_L - 2Q_{\ell_j} S_W^2 \right] \ell_j Z^\mu \\ &+ \frac{g'}{2T_W} \overline{\ell}_j \gamma_\mu \left[(-2T_8 - \beta T_W^2 \Lambda_3) P_L + 2\beta Q_{\ell_j} T_W^2 P_R \right] \ell_j Z^{\mu'} \Big\} \\ &+ \sum_{m=1}^2 \frac{g'}{2T_W} \overline{q}_m \gamma_\mu \left[(2T_8 + \beta Q_{q_m} T_W^2 \Lambda_1) P_L + 2\beta Q_{q_m} T_W^2 P_R \right] q_m Z^{\mu'} \\ &+ \frac{g'}{2T_W} \overline{q}_3 \gamma_\mu \left[(-2T_8 + \beta Q_{q_3} T_W^2 \Lambda_2) P_L + 2\beta Q_{q_3} T_W^2 P_R \right] q_3 Z^{\mu'}, \end{aligned} \quad (6)$$

where Q_j with $j = 1, 2, 3$ has been written in a SM-like notation i.e. it refers to triplets of quarks associated with the three generations of quarks (SM does not make difference in the family representations). On the other hand, the coupling of the exotic gauge boson (Z'_μ) with the two former families are different from the ones involving the third family. This is because the third family transforms differently as it was remarked in table 1. Consequently, there are terms where only the components $m = 1, 2$ are summed, leaving the third one in a term apart. Q_{q_j} are the electric charges. The Gell-Mann matrices $T_3 = \frac{1}{2}diag(1, -1, 0)$ and $T_8 = \frac{1}{2\sqrt{3}}diag(1, 1, -2)$ are introduced in the notation. We also define $\Lambda_1 = diag(-1, \frac{1}{2}, 2)$, $\Lambda_2 = diag(\frac{1}{2}, -1, 2)$ and the projectors $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. Finally, ℓ_j denote the leptonic triplets with Q_{ℓ_j} denoting their electric charges and $\Lambda_3 = diag(1, 1, 2Q_1)$ with $Q_1 = 0, -1$ for $\beta = -\frac{1}{\sqrt{3}}, -\sqrt{3}$ respectively.

The neutral lagrangian (6) can be written as

$$\begin{aligned} \mathcal{L}^{NC} = & \frac{g}{2C_W} \left\{ \sum_{j=1}^3 \overline{Q_j} \gamma_\mu \left[g_V^{Q_j} - g_A^{Q_j} \gamma_5 \right] Q_j Z^\mu + \overline{\ell_j} \gamma_\mu \left[g_V^{\ell_j} - g_A^{\ell_j} \gamma_5 \right] \ell_j Z^\mu \right. \\ & + \overline{\ell_j} \gamma_\mu \left[\tilde{g}_V^{\ell_j} - \tilde{g}_A^{\ell_j} \gamma_5 \right] \ell_j Z^{\mu'} + \sum_{m=1}^2 \overline{q_m} \gamma_\mu \left[\tilde{g}_V^{q_m} - \tilde{g}_A^{q_m} \gamma_5 \right] q_m Z^{\mu'} \\ & \left. + \overline{q_3} \gamma_\mu \left[\tilde{g}_V^{q_3} - \tilde{g}_A^{q_3} \gamma_5 \right] q_3 Z^{\mu'} \right\}, \end{aligned} \quad (7)$$

with the vector and axial vector couplings given by

$$\begin{aligned} g_V^f &= T_3 - 2Q_f S_W^2, \quad g_A^f = T_3 \\ \tilde{g}_{V,A}^{q_m} &= \frac{C_W^2}{\sqrt{1 - (1 + \beta^2) S_W^2}} \left[T_8 + \beta Q_{q_m} T_W^2 \left(\frac{1}{2} \Lambda_1 \pm 1 \right) \right] \\ \tilde{g}_{V,A}^{q_3} &= \frac{C_W^2}{\sqrt{1 - (1 + \beta^2) S_W^2}} \left[-T_8 + \beta Q_{q_3} T_W^2 \left(\frac{1}{2} \Lambda_2 \pm 1 \right) \right] \\ \tilde{g}_{V,A}^{\ell_j} &= \frac{C_W^2}{\sqrt{1 - (1 + \beta^2) S_W^2}} \left[-T_8 - \beta T_W^2 \left(\frac{1}{2} \Lambda_3 \mp Q_{\ell_j} \right) \right], \end{aligned} \quad (8)$$

where $f = Q_j, \ell_j$ in the first line. It is noted that $g_{V,A}^f$ are the same as the SM definitions and $\tilde{g}_{V,A}^f$ are new β -dependent couplings of Z'_μ (i.e. model dependent), whose values also depend on the family realization from table 2. On the other hand, there is a small mixing angle between Z_μ and Z'_μ given by Eq. (5), where $Z_{1\mu}$ is the SM-like neutral boson and $Z_{2\mu}$ the exotic ones. Taking a very small angle, we can do $C_\theta \simeq 1$ so that the lagrangian (7) becomes

$$\begin{aligned} \mathcal{L}^{NC} = & \sum_{j=1}^3 \left\{ \frac{g}{2C_W} \overline{Q_j} \gamma_\mu \left[G_V^{Q_j} - G_A^{Q_j} \gamma_5 \right] Q_j Z_1^\mu + \frac{g}{2C_W} \overline{\ell_j} \gamma_\mu \left[G_V^{\ell_j} - G_A^{\ell_j} \gamma_5 \right] \ell_j Z_1^\mu \right. \\ & + \frac{g}{2C_W} \overline{Q_j} \gamma_\mu \left[\tilde{G}_V^{Q_j} - \tilde{G}_A^{Q_j} \gamma_5 \right] Q_j Z_2^\mu + \frac{g}{2C_W} \overline{\ell_j} \gamma_\mu \left[\tilde{G}_V^{\ell_j} - \tilde{G}_A^{\ell_j} \gamma_5 \right] \ell_j Z_2^\mu \left. \right\}, \end{aligned} \quad (9)$$

where the couplings associated with $Z_{1\mu}$ are

$$\begin{aligned} G_{V,A}^f &= g_{V,A}^f + \delta g_{V,A}^f, \\ \delta g_{V,A}^f &= \tilde{g}_{V,A}^f S_\theta, \end{aligned} \quad (10)$$

and the couplings associated with $Z_{2\mu}$ are

$$\begin{aligned} \tilde{G}_{V,A}^f &= \tilde{g}_{V,A}^f - \delta \tilde{g}_{V,A}^f, \\ \delta \tilde{g}_{V,A}^f &= g_{V,A}^f S_\theta. \end{aligned} \quad (11)$$

4 Z-Pole Observables

The couplings of the $Z_{1\mu}$ in eq. (9) have the same form as the SM neutral couplings but by replacing the vector and axial vector couplings $g_{V,A}^{SM}$ by $G_{V,A} = g_{V,A}^{SM} + \delta g_{V,A}$, where $\delta g_{V,A}$ (given by eq. (10)) is a correction due to the small $Z_\mu - Z'_\mu$ mixing angle θ . For this reason all the analytical parameters at the Z pole have the same SM-form but with small correction factors that depend on the family assignment. The partial decay widths of Z_1 into fermions $f\bar{f}$ is described by [16, 17]:

$$\Gamma_f^{SM} = \frac{N_c^f G_f M_{Z_1}^3}{6\sqrt{2}\pi} \rho_f \left[\frac{3\beta_K - \beta_K^3}{2} \left(g_V^f \right)^2 + \beta_K^3 \left(g_A^f \right)^2 \right] R_{QED} R_{QCD}, \quad (12)$$

where $N_c^f = 1, 3$ for leptons and quarks respectively, $R_{QED, QCD}$ are global final-state QED and QCD corrections, and $\beta_K = \sqrt{1 - \frac{4m_b^2}{M_Z^2}}$ considers kinematic corrections only important for the b -quark. Universal electroweak corrections sensitive to the top quark mass are taken into account in $\rho_f = 1 + \rho_t$ and in g_V^{SM} which is written in terms of an effective Weinberg angle [16]

$$\overline{S_W}^2 = \kappa_f S_W^2 = \left(1 + \frac{\rho_t}{T_W^2} \right) S_W^2, \quad (13)$$

with $\rho_t = 3G_f m_t^2 / 8\sqrt{2}\pi^2$. Non-universal vertex corrections are also taken into account in the $Z_1 \bar{b}b$ vertex with additional one-loop leading terms given by [16, 17]

$$\rho_b \rightarrow \rho_b - \frac{4}{3}\rho_t \text{ and } \kappa_b \rightarrow \kappa_b + \frac{2}{3}\rho_t. \quad (14)$$

Table 3 resumes some observables, with their experimental values from CERN collider (LEP), SLAC Liner Collider (SLC) and data from atomic parity violation [16], the SM predictions and the expressions predicted by 331 models. We use $M_{Z_1} = 91.1876 \text{ GeV}$, $m_t = 176.9 \text{ GeV}$, $S_W^2 = 0.2314$, and for m_b we use [18]

$$\overline{m}_b(\mu \rightarrow M_{Z_1}) = m_b \left[1 + \frac{\alpha_S(\mu)}{\pi} \left(\ln \frac{m_b^2}{\mu^2} - \frac{4}{3} \right) \right],$$

with $m_b \approx 4.5$ GeV the pole mass, $\overline{m}_b(\mu \rightarrow M_{Z_1})$ the running mass at M_{Z_1} scale in the \overline{MS} scheme, and $\alpha_S(M_{Z_1}) = 0.1213 \pm 0.0018$ the strong coupling constant.

The 331 predictions from table 3 are expressed in terms of SM values corrected by

$$\begin{aligned}\delta_Z &= \frac{\Gamma_u^{SM}}{\Gamma_Z^{SM}}(\delta_u + \delta_c) + \frac{\Gamma_d^{SM}}{\Gamma_Z^{SM}}(\delta_d + \delta_s) + \frac{\Gamma_b^{SM}}{\Gamma_Z^{SM}}\delta_b + 3\frac{\Gamma_\nu^{SM}}{\Gamma_Z^{SM}}\delta_\nu + 3\frac{\Gamma_e^{SM}}{\Gamma_Z^{SM}}\delta_\ell; \\ \delta_{had} &= R_c^{SM}(\delta_u + \delta_c) + R_b^{SM}\delta_b + \frac{\Gamma_d^{SM}}{\Gamma_{had}^{SM}}(\delta_d + \delta_s); \\ \delta_\sigma &= \delta_{had} + \delta_\ell - 2\delta_Z; \\ \delta A_f &= \frac{\delta g_V^f}{g_V^f} + \frac{\delta g_A^f}{g_A^f} - \delta_f,\end{aligned}\tag{15}$$

where for the light fermions

$$\delta_f = \frac{2g_V^f\delta g_V^f + 2g_A^f\delta g_A^f}{\left(g_V^f\right)^2 + \left(g_A^f\right)^2},\tag{16}$$

while for the b -quark

$$\delta_b = \frac{(3 - \beta_K^2)g_V^b\delta g_V^b + 2\beta_K^2g_A^b\delta g_A^b}{\left(\frac{3 - \beta_K^2}{2}\right)\left(g_V^b\right)^2 + \beta_K^2\left(g_A^b\right)^2}.\tag{17}$$

The above expressions are evaluated in terms of the effective Weinberg angle from eq. (13).

For the predicted SM partial decay given by (12), we use the following values taken from ref. [16]

$$\begin{aligned}\Gamma_u^{SM} &= 0.3004 \pm 0.0002 \text{ GeV}; & \Gamma_d^{SM} &= 0.3832 \pm 0.0002 \text{ GeV}; \\ \Gamma_b^{SM} &= 0.3758 \pm 0.0001 \text{ GeV}; & \Gamma_\nu^{SM} &= 0.16729 \pm 0.00007 \text{ GeV}; \\ \Gamma_e^{SM} &= 0.08403 \pm 0.00004 \text{ GeV}.\end{aligned}$$

The weak charge is written as

$$Q_W = Q_W^{SM} + \Delta Q_W = Q_W^{SM}(1 + \delta Q_W),\tag{18}$$

where $\delta Q_W = \frac{\Delta Q_W}{Q_W^{SM}}$. The deviation ΔQ_W is [19]

$$\Delta Q_W = \left[\left(1 + 4 \frac{S_W^4}{1 - 2S_W^2} \right) Z - N \right] \Delta \rho_M + \Delta Q'_W,\tag{19}$$

and $\Delta Q'_W$ which contains new physics gives

$$\begin{aligned}\Delta Q'_W &= -16 \left[(2Z + N) \left(g_A^e \tilde{g}_V^u + \tilde{g}_A^e g_V^u \right) + (Z + 2N) \left(g_A^e \tilde{g}_V^d + \tilde{g}_A^e g_V^d \right) \right] S_\theta \\ &\quad - 16 \left[(2Z + N) \tilde{g}_A^e \tilde{g}_V^u + (Z + 2N) \tilde{g}_A^e \tilde{g}_V^d \right] \frac{M_{Z_1}^2}{M_{Z_2}^2}.\end{aligned}\tag{20}$$

Quantity	Experimental Values	Standard Model	331 Model
$\Gamma_Z [GeV]$	2.4952 ± 0.0023	2.4972 ± 0.0012	$\Gamma_Z^{SM} (1 + \delta_Z)$
$\Gamma_{had} [GeV]$	1.7444 ± 0.0020	1.7435 ± 0.0011	$\Gamma_{had}^{SM} (1 + \delta_{had})$
$\Gamma_{(\ell^+\ell^-)} MeV$	83.984 ± 0.086	84.024 ± 0.025	$\Gamma_{(\ell^+\ell^-)}^{SM} (1 + \delta_\ell)$
$\sigma_{had} [nb]$	41.541 ± 0.037	41.472 ± 0.009	$\sigma_{had}^{SM} (1 + \delta_\sigma)$
R_e	20.804 ± 0.050	20.750 ± 0.012	$R_e^{SM} (1 + \delta_{had} + \delta_e)$
R_μ	20.785 ± 0.033	20.751 ± 0.012	$R_\mu^{SM} (1 + \delta_{had} + \delta_\mu)$
R_τ	20.764 ± 0.045	20.790 ± 0.018	$R_\tau^{SM} (1 + \delta_{had} + \delta_\tau)$
R_b	0.21638 ± 0.00066	0.21564 ± 0.00014	$R_b^{SM} (1 + \delta_b - \delta_{had})$
R_c	0.1720 ± 0.0030	0.17233 ± 0.00005	$R_c^{SM} (1 + \delta_c - \delta_{had})$
A_e	0.15138 ± 0.00216	0.1472 ± 0.0011	$A_e^{SM} (1 + \delta A_e)$
A_μ	0.142 ± 0.015	0.1472 ± 0.0011	$A_\mu^{SM} (1 + \delta A_\mu)$
A_τ	0.136 ± 0.015	0.1472 ± 0.0011	$A_\tau^{SM} (1 + \delta A_\tau)$
A_b	0.925 ± 0.020	0.9347 ± 0.0001	$A_b^{SM} (1 + \delta A_b)$
A_c	0.670 ± 0.026	0.6678 ± 0.0005	$A_c^{SM} (1 + \delta A_c)$
A_s	0.895 ± 0.091	0.9357 ± 0.0001	$A_s^{SM} (1 + \delta A_s)$
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01626 ± 0.00025	$A_{FB}^{(0,e)SM} (1 + 2\delta A_e)$
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013	0.01626 ± 0.00025	$A_{FB}^{(0,\mu)SM} (1 + \delta A_e + \delta A_\mu)$
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017	0.01626 ± 0.00025	$A_{FB}^{(0,\tau)SM} (1 + \delta A_e + \delta A_\tau)$
$A_{FB}^{(0,b)}$	0.0997 ± 0.0016	0.1032 ± 0.0008	$A_{FB}^{(0,b)SM} (1 + \delta A_e + \delta A_b)$
$A_{FB}^{(0,c)}$	0.0706 ± 0.0035	0.0738 ± 0.0006	$A_{FB}^{(0,c)SM} (1 + \delta A_e + \delta A_c)$
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1033 ± 0.0008	$A_{FB}^{(0,s)SM} (1 + \delta A_e + \delta A_s)$
$Q_W(Cs)$	-72.69 ± 0.48	-73.19 ± 0.03	$Q_W^{SM} (1 + \delta Q_W)$

Table 3: The parameters for experimental values, SM predictions and 331 corrections. The values are taken from ref. [16]

$\beta = -\frac{1}{\sqrt{3}}$	
A, B	$\Delta Q'_W = (4.68Z + 2.98N) S_\theta + (3.11Z + 4.00N) \frac{M_{Z_1}^2}{M_{Z_2}^2}$
C	$\Delta Q'_W = -(8.125Z + 9.82N) S_\theta - (5.78Z + 4.89N) \frac{M_{Z_1}^2}{M_{Z_2}^2}$
$\beta = -\sqrt{3}$	
A, B	$\Delta Q'_W = (2.81Z + 1.26N) S_\theta + (5.85Z + 42.25N) \frac{M_{Z_1}^2}{M_{Z_2}^2}$
C	$\Delta Q'_W = -(36.23Z + 37.785N) S_\theta - (115.04Z + 78.645N) \frac{M_{Z_1}^2}{M_{Z_2}^2}$

Table 4: *New physics contributions to ΔQ_W for the two 331 models according to the family assignment from table 2.*

Quarks Rep.	M_{Z_2} (GeV)	$S_\theta (\times 10^{-3})$
Rep. A	$M_{Z_2} \geq 1400$	$-0.9 \leq S_\theta \leq 2$
Rep. B	$M_{Z_2} \geq 1400$	$-0.9 \leq S_\theta \leq 2$
Rep. C	$M_{Z_2} \geq 2100$	$-0.9 \leq S_\theta \leq 0.9$

Table 5: *Bounds with $\beta = -\frac{1}{\sqrt{3}}$ for M_{Z_2} and S_θ for three quark representations at 95% CL*

For Cesium we have $Z = 55$, $N = 78$, and for the first term in (19) we take the value $\left[\left(1 + 4 \frac{S_W^4}{1-2S_W^2} \right) Z - N \right] \Delta \rho_M \simeq -0.01$ [19]. With the definitions in eq. (8) for $\beta = -\frac{1}{\sqrt{3}}$ and $-\sqrt{3}$, we displays in table 4 the new physics corrections to ΔQ_W given by eq. (20) for each representation of quarks listed in table 2. We get the same correction for the spectrum A and B due to the fact that the weak charge depends mostly on the up-down quarks, and A, B -cases maintain the same representation for this family.

With the expressions for the Z-pole observables and the experimental data shown in table 3, we perform a χ^2 fit for each representation A, B and C at 95% CL, which will allow us to display the family dependence in the model. We find the best allowed region in the plane $M_{Z_2} - S_\theta$. Figure 1 displays three cases for $\beta = -\frac{1}{\sqrt{3}}$, each one corresponding to the family representations from table 2 respectively, exhibiting family-dependent regions. The bounds for M_{Z_2} and S_θ are shown in table 5 for each family-choices. We can see that the lowest bound for M_{Z_2} is about 1400 GeV for models A and B , while model C with the lightest family defined in the third multiplet increases this bound to 2100 GeV. It is also noted that the A and B representations yield broader allowed regions for the mixing angle than model C , showing that the family choices is a fundamental issue in the phenomenology of 331 models. A -region and B -region are very similar because they present the same weak corrections, as it is shown in table 4; the small differences arise mostly due to the bottom correction in eq. (17). The table 4 also shows that the C -region contribute to the new physics correction with a contrary sign respect A and B corrections.

For $\beta = -\sqrt{3}$, we get the regions in figure 2, which also compares the three family choices. This model increases the lowest bound in the M_{Z_2} value to 4000 GeV for the A

Quarks Rep.	M_{Z_2} (GeV)	$S_\theta (\times 10^{-4})$
Rep. A	$M_{Z_2} \geq 4000$	$-1.2 \leq S_\theta \leq 1.7$
Rep. B	$M_{Z_2} \geq 4000$	$-1.2 \leq S_\theta \leq 1.7$
Rep. C	$M_{Z_2} \geq 10000$	$-1.2 \leq S_\theta \leq 1.2$

Table 6: *Bounds with $\beta = -\sqrt{3}$ for M_{Z_2} and S_θ for three quark representations at 95% CL*

and B spectrums, and to 10000 GeV for the C spectrum. In this case A and B regions are equivalent. The table 6 displays the bounds for the mixing angle and M_{Z_2} . We can see that the mixing angle is smaller in about one order of magnitude than the angle predicted by $\beta = -\frac{1}{\sqrt{3}}$ model. These differences between both models arise mostly because they present variations to the weak charge corrections (see table 4).

5 Conclusions

The $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models for three families with $\beta = -\frac{1}{\sqrt{3}}$ and $-\sqrt{3}$, corresponding to the Long [9] and Pisano-Pleitez-Frampton bilepton models [8] respectively, were studied under the framework of family dependence.

As it is shown in table 2, we found three different assignments of quarks into the mass family basis. Each assignment determines different weak couplings of the quarks to the extra neutral current associated to Z_2 , which holds a small angle mixing with respect to the SM-neutral current associated to Z_1 . This mixing gives different allowed regions in the $M_{Z_2} - S_\theta$ plane for the LEP parameters at the Z-pole and including data from the atomic parity violation.

Performing a χ^2 fit at 95% CL we found for the Long model that $M_{Z_2} \geq 1400$ when $-0.9 \times 10^{-3} \leq S_\theta \leq 2 \times 10^{-3}$ for A and B assignments, and $M_{Z_2} \geq 2100$ when $-0.9 \times 10^{-3} \leq S_\theta \leq 0.9 \times 10^{-3}$ for C assignment. For the Pleitez model we got $M_{Z_2} \geq 4000$ for $-1.2 \times 10^{-4} \leq S_\theta \leq 1.7 \times 10^{-4}$, and $M_{Z_2} \geq 10000$ for $-1.2 \times 10^{-4} \leq S_\theta \leq 1.2 \times 10^{-4}$ for each case.

Unlike the SM where the family assignment is arbitrary without any phenomenological change, our results show how this assignment yields differences in the numerical predictions for two 331 models. We see that the lowest bound for M_{Z_2} is higher than those obtained by other authors for one family models [7]. Due to the restriction of the data from the atomic parity violation, we are getting a differences of about one order of magnitude in the lowest bound for the M_{Z_2} . In addition we found that the bounds associated with the angle mixing is highly suppressed ($\sim 10^{-4}$) in the Pleitez model when the lightest quarks family transform differently respect the two heavier families.

This study can be extended if we consider linear combinations among the three family assignments according to the ansatz of the quarks mass matrix in agreement with the physical mass and mixing angle mass. In this case, the allowed regions for M_{Z_2} Vs S_θ would be a combination among the regions obtained for A , B and C models.

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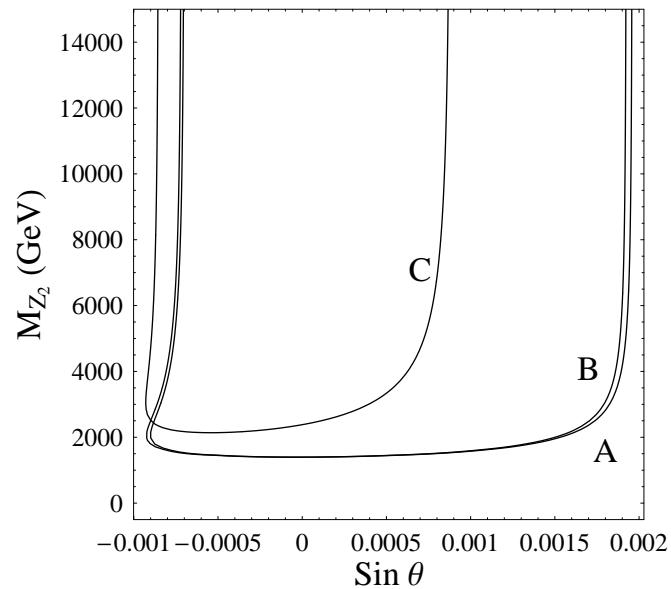


Figure 1: The allowed region for M_{Z_2} vs $\sin \theta$ in the model with $\beta = -1/\sqrt{3}$. A, B and C correspond to the assignment of families from table 2

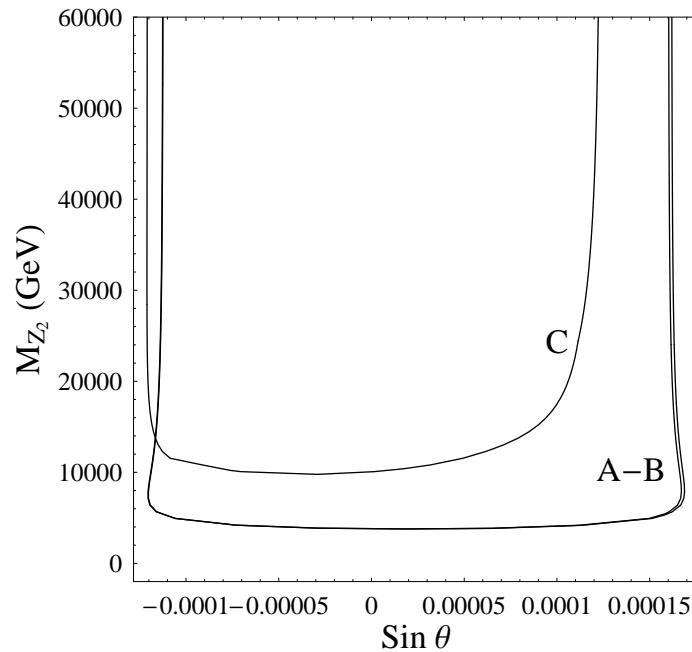


Figure 2: The allowed region for M_{Z_2} vs $\sin \theta$ in the model with $\beta = -\sqrt{3}$. A, B and C correspond to the assignment of families from table 2